A Principle-Based Analysis of Bipolar Argumentation Semantics

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Abstract. In this paper, we introduce and study seven types of semantics for bipolar argumentation frameworks, each extending Dung's interpretation of attack with a distinct interpretation of support. First, we introduce three types of defence-based semantics by adapting the notions of defence. Second, we examine two types of selection-based semantics that select extensions by counting the number of supports. Third, we analyse two types of traditional reduction-based semantics under deductive and necessary interpretations of support. We provide full analysis of twenty-eight bipolar argumentation semantics and ten principles.

Keywords: Bipolar argumentation semantics · Support · Principle-based approach · Knowledge representation and reasoning.

1 Introduction

In this paper, we consider so-called bipolar argumentation frameworks [13,14,15] containing not only attacks but also supports among arguments. While there is general agreement in the formal argumentation literature on how to interpret attack, even when different kinds of semantics have been defined, there is much less consensus on how to interpret support [18]. There exist very few results and studies about the role of support in abstract argumentation. Consequently, the principle-based approach is used to bring structure to the field [16,42]. In this paper, we address the following research questions: In which ways can support affect attack, defence and argumentation semantics? Which principles can be introduced to distinguish between, and characterise, these semantics?

There exist different approaches to extending Dung's abstract theory by taking into consideration the support relation. The relation between support and attack has been studied extensively in reduction-based approaches, in the sense that deductive and necessary interpretations of support give rise to various notions of indirect attack [16], thus, they typically give opposite results. Deductive support [8] captures the intuition that if

L. Yu and C. Al Anaissy contributed equally to this work.

a supports b, then the acceptance of a implies the acceptance of b. This intuition is characterised by the so-called closure principle [16]. Necessary support [29] captures the intuition that if a supports b, then the acceptance of a is necessary to obtain the acceptance of b, or equivalently, the acceptance of b implies the acceptance of a. It has been characterised by the inverse closure principle [33]. Another approach to handling support is the evidence-based approach [31] where the notion of evidential support is introduced. An argument cannot stand unless it is supported by evidential support. Support can also be seen as an inference relation between the premises and the conclusion of the argument itself [35]. Moreover, in selection-based approaches [23], support is used only to select some of the extensions provided in Dung's semantics, and thus does not change the definition of attack, or defence.

Despite the relevance and significance of all the mentioned approaches, we think that there is still the need to explore other approaches that have not been yet considered for bipolar argumentation frameworks. The aim of our research is not to replace other approaches but rather to point to the existence of other interesting ones that can be applied depending on the chosen application. Note that our approach is novel in its methodology. On one hand, reduction-based approaches can be seen as a kind of pre-processing step for Dung's theory of abstract argumentation (i.e. adding the complex attacks and then applying Dung's semantics). On the other hand, selection-based approaches can be seen as a kind of post-processing step (i.e. applying Dung's semantics and then applying the approach to select some of the extensions). Differently from those two groups of approaches, our approach (i.e. the defence-based approach) does not affect the concept of attack and conflict-freeness, but rather changes the definition of defence.

Most of the principles we introduce and use for analysing bipolar argumentation are in the same spirit as the principles used in the principle-based analysis of Dung's semantics [40]. For example, the robustness of argumentation semantics when adding or removing attacks plays a central role [39]. In this paper, we consider robustness when adding or removing support relations. We also introduce some principles specifically defined for support, such as to which extent an argument is accepted while receiving support from others.

The layout of this paper is as follows. We first introduce three defence-based semantics, then two selection-based ones, and we study two traditional reduction-based ones. Then, we introduce ten principles, and we analyse which properties are satisfied by which semantics, before concluding and introducing the ideas for future work.

2 Bipolar Argumentation Framework

Bipolar argumentation frameworks extend the argumentation frameworks introduced by Dung (1995) with a binary support relation among the arguments.

Definition 1 (Bipolar Argumentation Framework [15]). *A bipolar argumentation framework (BAF) is a triple* $\langle Ar, att, sup \rangle$ *where* Ar *is a finite set called arguments, and* $att, sup \subseteq Ar \times Ar$ *are binary relations over* Ar *called attack and support respectively.*

Figure 1 illustrates three BAFs, where attack relations are depicted by solid arrows, and support relations are depicted by dashed arrows. Given a, b in Ar, $(a,b) \in att$ standing for a attacks b, and $(a,b) \in sup$ standing for a supports b, the definitions of conflict-freeness and defence provided by Dung are called conflict-free$_0$ and defended$_0$.

Definition 2 (Conflict-free$_0$ and Defended$_0$ [21]). *Let $\mathcal{F} = \langle Ar, att, sup \rangle$ be a BAF. A set of arguments $E \subseteq Ar$ is conflict-free$_0$, written as $cf_0(\mathcal{F}, E)$, iff there are no arguments a and b in E such that a attacks b. The set of arguments defended$_0$ by E, written as $d_0(\mathcal{F}, E)$, is the set of a arguments such that for every argument b attacking a, there is an argument c in E attacking b.*

2.1 Defence-based semantics

We first define three new types of defence-based semantics, which are based on conflict-free$_0$ and the new definitions of defended$_1$, defended$_2$ and defended$_3$. To have a generic definition of defence-based semantics (Definition 5), we also define conflict-free$_1$, conflict-free$_2$, and conflict-free$_3$, for each of the new types of semantics. The three notions of defended have stronger requirements than defended$_0$. Defended$_1$ requires that the argument defending$_0$ another argument also supports it. Defended$_2$ requires that a defender is supported. Moreover, defended$_3$ requires not only that the attackers are attacked, but also that all supporters of the attackers are attacked as well.

Definition 3 (Conflict-free$_{1-3}$ and Defended$_{1-3}$). *Let $\mathcal{F} = \langle Ar, att, sup \rangle$ be a BAF. We use the same definition as Dung for conflict-free, i.e. $cf_1 \equiv cf_2 \equiv cf_3 \equiv cf_0$. Moreover:*

- *the set of arguments defended$_1$ by E, written as $d_1(\mathcal{F}, E)$, is the set of arguments a in Ar such that for each argument b in Ar attacking a, there exists an argument c in E attacking b and supporting a (supporting-defence);*
- *the set of arguments defended$_2$ by E, written as $d_2(\mathcal{F}, E)$, is the set of arguments a in Ar such that for all arguments b in Ar attacking a, there exists an argument c in E attacking b, and there is an argument d in E supporting c (supported-defence);*
- *the set of arguments defended$_3$ by E, written as $d_3(\mathcal{F}, E)$, is the set of arguments a in Ar such that for all arguments b in Ar attacking a, there exists an argument c in E attacking b, and for all arguments d in Ar supporting b, there is an argument e in E attacking d (attacking-defence).*

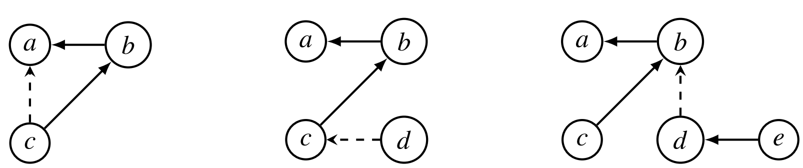


Fig. 1: Three BAFs illustrating the three new defence notions, for the lefthand figure, $d_1(\mathcal{F}, \{c\}) = \{a, c\}$; for the middle figure, $d_2(\mathcal{F}, \{c, d\}) = \{a, c, d\}$; for the righthand figure, $d_3(\mathcal{F}, \{c, e\}) = \{a, c, e\}$

Following Dung's approach,we say the characteristic function $d_i(\mathcal{F},E)$ of a bipolar argumentation framework BAF is as follows:

- $d_i(\mathcal{F},E) : 2^{Ar} \to 2^{Ar}$,
- $d_i(\mathcal{F},E) = \{A \,|\, A$ is defended$_i$ by $E\}$, for $i \in \{0,1,2,3\}$.

Definition 4 (Admissibility$_{0-3}$). *A set of arguments E in BAF $\mathcal{F} = \langle Ar, att, sup\rangle$, is said to be admissible$_i$ iff E is conflict-free$_i$ and $E \subseteq d_i(\mathcal{F},E)$, for $i \in \{0,1,2,3\}$.*

To define the complete (abbreviated as c), preferred (p), and stable (s) semantics of bipolar argumentation frameworks, the following definition is generic and can be used with any kind of conflict-freeness and defence.

Definition 5 (Semantics$_{0-3}$). *An extension-based semantics σ is a function that maps a BAF $\mathcal{F} = \langle Ar, att, sup\rangle$ onto a set of subsets of Ar, written as $\sigma_i^x(\mathcal{F})$, where $i \in \{0,1,2,3\}$, $x \in \{c,p,s\}$ as follows:*

- $\sigma_i^c(\mathcal{F}) = \{E \subseteq Ar \mid cf_i(\mathcal{F},E)$ *and* $d_i(\mathcal{F},E) = E\}$;
- $\sigma_i^p(\mathcal{F}) = \{E \subseteq Ar \mid E$ *is admissible$_i$, and for all admissible$_i$ set* $E', E \nsubseteq E'\}$;
- $\sigma_i^s(\mathcal{F}) = \{E \subseteq Ar \mid E$ *is admissible$_i$, and for all arguments a not in E, there is an argument b in E attacking a}.*

Most of the following propositions were introduced and proved for semantics$_0$ by Dung (1995). We prove that the above three new defence semantics are able to conserve the relations among complete$_i$, preferred$_i$, and grounded$_i$ for $i \in \{1,2,3\}$ and stable$_i$ for $i = 3$.

Lemma 1 (Fundamental Lemma). *Let E be an admissible$_i$ set of arguments, and A_1 and A_2 be two arguments which are defended$_i$ by E. Then for $i \in \{0,1,2,3\}$, we have the following:*

- $E' = E \cup \{A_1\}$ *is admissible$_i$.*
- A_2 *is defended$_i$ by* E'.

The following theorem follows directly from the Fundamental Lemma.

Theorem 1. *Let $\mathcal{F}$ be a BAF, for $i \in \{0,1,2,3\}$:*

- *The set of all admissible$_i$ sets of $\mathcal{F}$ forms a complete partial order with respect to set inclusion.*
- *For each admissible$_i$ set S of $\mathcal{F}$, there exists a preferred$_i$ extension E of $\mathcal{F}$ such that $S \subseteq E$.*

Note that the empty set is always admissible$_i$, we have the following Corollary for $i \in \{0,1,2,3\}$:

Corollary 1. *There exists at least one preferred$_i$ extension in any bipolar argumentation framework for $i \in \{0,1,2,3\}$.*

Proposition 1. *For $i \in \{0,1,2,3\}$, we have the following: every complete$_i$ extension is also admissible$_i$; every preferred$_i$ extension is also complete$_i$; every stable$_i$ extension is also preferred$_i$.*

Proposition 2. *The characteristic function $d_i(\mathcal{F},E)$ is monotonic (with respect to set inclusion) for $i \in \{0,1,2,3\}$.*

Proposition 3. *Any BAF $\mathcal{F}$ induces a complete lattice which is the power set of all the arguments in $\mathcal{F}$. The characteristic function $d_i(\mathcal{F},E)$, $i \in \{0,1,2,3\}$, is monotonic (with respect to set inclusion). Therefore, from Knaster–Tarski theorem:*

- *The set of fixed points of $d_i(\mathcal{F},E)$ is a complete lattice.*
- *$d_i(\mathcal{F},E)$ has a unique least fixed point which can be obtained either by doing the intersection of all the fixed points of $d_i(\mathcal{F},E)$, or by iteratively applying $d_i(\mathcal{F},E)$, to the empty set.*

Definition 6 (Grounded$_{0-3}$ semantics). *The grounded$_i$ extension of a BAF $\mathcal{F} = \langle Ar, att, sup\rangle$, is the least fixed point of the characteristic function $d_i(\mathcal{F},E)$, for $i \in \{0,1,2,3\}$. We denote the grounded$_i$ semantics by $\sigma_i^g(\mathcal{F})$.*

Proposition 4. *The grounded$_i$ extension of $\mathcal{F}$ for $i \in \{0,1,2,3\}$ is the minimal (with respect to set inclusion) complete extension of $\mathcal{F}$.*

We now give a real legal example to illustrate the intuition behind semantics$_1$. This example deals with a neighbor's quarrel over a row of conifers and was used to explain how the judge defends the claimant's interest [32].

Example 1 (Neighbours' quarrel over conifers). (...) The defendant argues that the conifers have been planted to reduce draught in his house, but this argument is absolutely unsound since most of the window posts are closed and the window that does open is located on a point higher than the tops of the conifers and has not been fitted with any anti-draught facilities. (...) Whereas the defendant has no considerable interest in these conifers, removal is of significant concern to the claimant since they block his view and take away the light. (...) (2981. Country court Enschede 6 October 1988)

The judge defends the standpoint that the claimant's interest in the removal of the conifers is greater than the defendant's interest in leaving them untouched. In the judge's preceding remarks, he mentions the defendant's argument: he does have a considerable interest in the conifers since they reduce draught in his house, thus he wants to keep the conifers. To support the standpoint of the claimant and against the defendant, the judge argues that the conifers block the view and take away the light, most of the window posts are closed and the opening window, which has no anti-draught facilities whatsoever, is located higher than the tops of the conifers.

As stated by Plug: "the judge's argumentation consists of a pro-argument and the refutation of a counter-argument which, in conjunction, form sufficient support for his standpoint." This type of defence inspires semantics$_1$.

We now give an example to illustrate the intuition behind semantics$_2$.

Example 2 (Twelve Angry Men play using Semantics$_2$). We consider an example extracted from the NoDE benchmark [10], which consists of annotated datasets extracted from a variety of sources (Debatepedia, Procon, Wikipedia web pages and the script of "Twelve Angry Men" play), where the aim of this benchmark is to analyse the support and attack relations between the arguments. We explore the Twelve Angry Men dataset, this play is about a jury consisting of twelve men who must decide whether a young man is guilty or not for murdering his father. Consider the following arguments extracted from this dataset.

– a_1: I think we proved that the old man couldn't have heard the boy say, "I'm going to kill you" but supposing he really did hear it? This phrase: how many times has each of you used it? Probably hundreds. "If you do that once more, Junior, I'm going to murder you." "Come on, Rocky, kill him!" We say it every day. This doesn't mean that we're going to kill someone.
– e_1: The phrase was "I'm going to kill you" and the kid screamed it out at the top of his lungs. Anybody says a thing like that the way he said it—they mean it.
– g_1: Do you really think the boy would shout out a ["I'm going to kill you"] so the whole neighbourhood would hear it? I don't think so. He's much too bright for that.

The example above is shown in Figure 2. In this example, argument g_1 attacks argument e_1 by raising some doubt about it. In the same manner, argument e_1 attacks argument a_1. We can see that the argument g_1 defends argument a_1 in Dung's sense. Just because argument g_1 is not attacked, argument a_1 is accepted.

In a legal case, any given argument must be evaluated based on the evidence provided to support it. In the absence of such evidence, the presence of at least a support, even if it is challenged, seems necessary. Therefore, one can ask themselves whether Dung's notion of defence seems enough, in this case, to say that the argument g_1 defends the argument a_1. Hence, for this kind of application, one might want to use a stronger notion of defence. An example of such a notion is our semantics$_2$, where an argument must be supported in order to be able to defend another argument. The idea behind this semantics is to provide a stronger and more restrictive defence notion than Dung's defence notion, by taking into account the support relation.

We consider now the following arguments extracted from the same dataset, to illustrate semantics$_2$.

– f: Maybe he didn't hear [the boy yelling "I'm going to kill you"]. I mean with the el noise.
– g: [The old man cannot be a liar, he must have heard the boy yelling "I'm going to kill you"].
– h: It stands to reason, [the old man can be a liar].
– i: Attention, maybe [the old man is a liar].

Contrary to the previous example, we see that argument i is supported by another one, hence it might be seen as having a better capacity to defend f. Formally, the set of arguments $\{h, i\}$ defends$_2$ the argument f.

Example 3 (Recruitment using semantics$_3$). Consider the following arguments.

– a: Alice should be hired as a professor.
– b: Alice lacks many essential qualifications to become a professor.
– c: Alice has few publications.
– d: Alice has recently got her PhD, she does not have enough teaching experience.

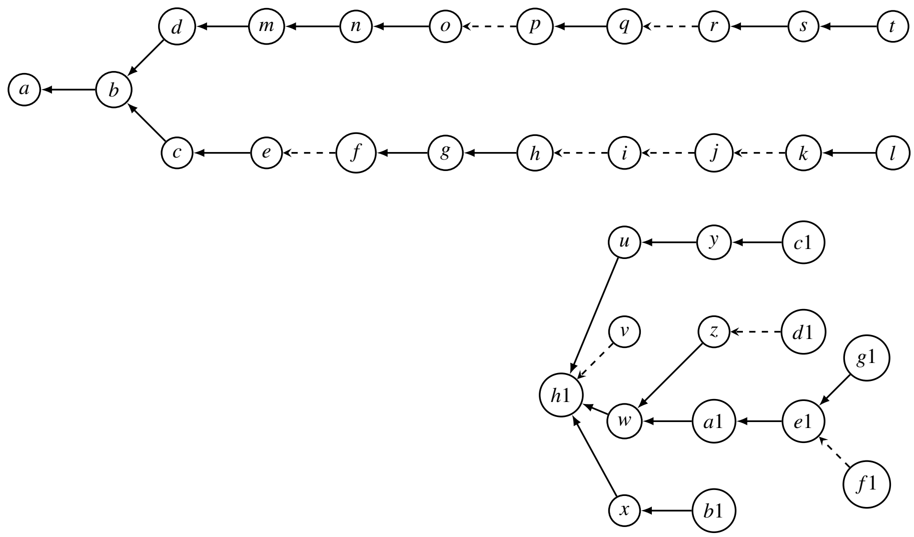


Fig. 2: The BAF illustrating the Twelve Angry Men dataset - Act 2.

- e: All of Alice's publications are in top conferences. When it comes to publications, quality beats quantity.
- f: Alice has taught 64 hours of practical works during every year of her PhD, which is considered enough as teaching experience.
- g: Alice is good at team work, she also has an excellent academic carrier, these two enable her to become a professor.

This example can be represented with the BAF depicted on the left-hand side of Figure 3. g fails to reinstate a because g does not attack b's supporters c and d. The set of arguments $\{e,g,f\}$ reinstates a because it attacks all the supporters of b. $\sigma_3^{c,g,p,s}(\mathcal{F}) = \{\{a,e,g,f\}\}$.

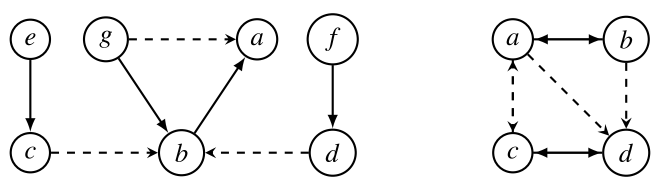


Fig. 3: A BAF illustrating recruitment case (on the left) and a BAF illustrating semantics$_4$ and semantics$_5$ (on the right)

2.2 Selection-based semantics

Support can be used in the post-processing step for Dung's theory of abstract argumentation [23]. Semantics$_4$ and semantics$_5$ are two selection-based approaches, i.e. they select extensions from semantics$_0$. Semantics$_4$ selects the extensions that have the largest number of internal supports, reflecting the idea that for a coalition, the more internal supports they have, the more cohesive they are. Semantics$_5$ selects the extensions that receive the most support from outside, reflecting the idea that the more support a coalition receives, the stronger it is. It thus interprets support as a kind of voting.

We say that argument b in E is internally supported if b receives support from arguments in E. Argument b in E is externally supported if b receives support from arguments that are outside E.

Definition 7 (Number of Internal and External Supports). *Let $\mathcal{F} = \langle Ar, att, sup\rangle$ be a BAF. For an extension $E \in \sigma_0^x$, the number of internal supports is written as NS_I, such that $NS_I(\mathcal{F}, E) = \mid \{(a,b) \in sup \mid a,b \in E\} \mid$, and the number of external supports is written as NS_O, such that $NS_O(\mathcal{F}, E) = \mid \{(a,b) \in sup \mid b \in E, a \in Ar \setminus E\} \mid$.*

Definition 8 (Semantics$_{4-5}$). *For each $\mathcal{F} = \langle Ar, att, sup\rangle$, for $x \in \{c, g, p, s\}$:*

- $\sigma_4^x(\mathcal{F}) = \arg\max_{E \in \sigma_0^x(\mathcal{F})} \{NS_I(\mathcal{F}, E)\}$*; and*
- $\sigma_5^x(\mathcal{F}) = \arg\max_{E \in \sigma_0^x(\mathcal{F})} \{NS_O(\mathcal{F}, E)\}$.

We use Example 4 to illustrate the difference between semantics$_4$ and semantics$_5$.

Example 4 (Semantics$_{4-5}$). Consider the bipolar argumentation framework on the right-hand side of Figure 3, $\sigma_0^c(\mathcal{F}) = \{\{a\},\{b\},\{c\},\{d\},\{a,d\},\{a,c\},\{b,d\},\{b,c\}\}$, $\sigma_0^{ps}(\mathcal{F}) = \{\{a,d\}, \{a,c\}, \{b,d\}, \{b,c\}\}$. Then, $\sigma_4^{cps}(\mathcal{F}) = \{\{a,c\}\}$, because $\{a,c\}$ has the biggest number of internal supports. Then, $\sigma_5^c(\mathcal{F}) = \{\{d\},\{a,d\}\}$, and $\sigma_5^{ps}(\mathcal{F}) = \{\{a,d\}\}$, because they receive the biggest number of external supports.

2.3 Reduction-based semantics

Reduction-based approaches have been studied extensively in the literature [13,14,15]. Semantics$_6$ and semantics$_7$ are two reduction-based approaches where support is used as pre-processing for Dung semantics. The corresponding abstract argumentation frameworks are reduced by adding indirect attacks from the interaction of attack and support with different interpretations, i.e. deductive support and necessary support. So-called supported attack and mediated attack come from the interplay between attack and deductive support, while secondary attack and extended attack come from the interplay between attack and necessary support.

Definition 9. *(Four Indirect Attacks [15]) Let $\mathcal{F} = \langle Ar, att, sup\rangle$ be a BAF, and let arguments $a, b, c \in Ar$. There is:*

- *a supported attack from a to b in $\mathcal{F}$ iff there exists an argument c such that there is a sequence of supports from a to c and c attacks b, represented as $(a,b) \in att^{supp}$;*

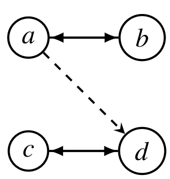
- *a mediated attack from a to b in $\mathcal{F}$ iff there exists an argument c such that there is a sequence of supports from b to c and a attacks c, represented as* $(a,b) \in att^{med}$;
- *a super-mediated attack from a to b in $\mathcal{F}$ iff there exists an argument c such that there is a sequence of supports from b to c and a directly or supported-attacks c, represented as* $(a,b) \in att^{med}_{att^{supp}}$;
- *a secondary attack from a to b in $\mathcal{F}$ iff there exists an argument c such that there is a sequence of supports from c to b and a attacks c, so that* $(a,b) \in att^{sec}$;
- *an extended attack from a to b in $\mathcal{F}$ iff there exists an argument c such that there is a sequence of supports from c to a and c attacks b, so that* $(a,b) \in att^{ext}$.

Definition 10 (Semantics$_{6-7}$ [15]). *Let* $\mathcal{F} = \langle Ar, att, sup \rangle$ *be a BAF:*

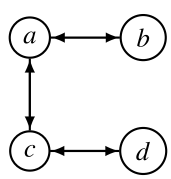
- *let* $att' = \{att^{supp}, att^{med}_{att^{supp}}\}$ *be the collection of supported and super-mediated attacks in $\mathcal{F}$, and we have* $RD(\mathcal{F}) = (Ar, att \cup \bigcup att')$, *and* $\sigma^x_6(\mathcal{F}) = \sigma^x_0(RD(\mathcal{F}))$;
- *let* $att' = \{att^{sec}, att^{ext}\}$ *be the collection of secondary and extended attacks in $\mathcal{F}$, and we have* $RN(\mathcal{F}) = (Ar, att \cup \bigcup att')$, *and* $\sigma^x_7(\mathcal{F}) = \sigma^x_0(RN(\mathcal{F}))$.

We use Example 5 to illustrate semantics$_6$ and semantics$_7$.

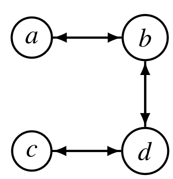
Example 5 (Semantics$_{6-7}$). Consider the bipolar argumentation framework in Figure 4.1. If the interpretation of support from a to d is deductive, a supported-attacks c, c mediated-attacks a. We have $RD(\mathcal{F}) = \langle Ar, att \cup \{(a,c),(c,a)\}\rangle$ as visualised in Figure 4.2. $\sigma^g_6 = \{\emptyset\}$, $\sigma^c_6 = \{\{b\},\{d\},\{a,d\}, \{b,d\},\{b,c\}\}$, and $\sigma^p_6 = \sigma^s_6 = \{\{a,d\},\{b,d\},\{b,c\}\}$. If the interpretation of support from a to d is necessary, then b secondary-attacks d, and d extended-attacks b. We have $RN(\mathcal{F}) = \langle Ar, att \cup \{\{b,d\},\{d,b\}\}\rangle$ as visualised in Figure 4.3. $\sigma^g_7 = \{\emptyset\}$, $\sigma^c_7 = \{\{a\},\{c\},\{a,d\},\{a,c\},\{b,c\}\}$, $\sigma^p_7 = \sigma^s_7 = \{\{a,d\},\{a,c\},\{b, c\}\}$.



4.1 A BAF $\mathcal{F}$



4.2 RD($\mathcal{F}$)



4.3 RN($\mathcal{F}$)

Fig. 4: Deductive and necessary interpretations give different corresponding AFs

3 Principles

In this section, we present ten principles. Due to the space limitation, we only present some interesting proofs, others can be found in additional supplement. The first principle concerns the support relation alone. It expresses transitivity of support.

Principle 1 (Transitivity) *A semantics σ_i^x for BAFs satisfies the transitivity principle iff for all BAFs $\mathcal{F} = \langle Ar, att, sup \rangle$, if a supports b, and b supports c, then $\sigma_i^x \langle Ar, att, sup \rangle = \sigma_i^x \langle Ar, att, sup \cup \{a, c\} \rangle$.*

Principle 2 states that supports can be used to select extensions.

Principle 2 (Extension Selection) *A semantics σ_i^x for BAFs satisfies the extension selection principle iff for all BAFs where $\mathcal{F} = \langle Ar, att, sup \rangle$, that $\sigma_i^x(Ar, att, sup) \subseteq \sigma_i^x(Ar, att, \emptyset)$.*

Principle 3 and Principle 4 are robustness principles that distinguish between semantics$_4$ and semantics$_5$. The set of robustness principles was proposed by Rienstra et al. [38]. Here, we adapt their idea to bipolar argumentation in order to investigate the robustness of bipolar argumentation semantics when removing and adding support. Principle 3 states that if two arguments a and b are in an extension E such that a supports b, then E is still an extension after we remove the support from a to b.

Principle 3 (Internal Support Removal Robustness) *A semantics σ_i^x for BAFs satisfies the internal support removal robustness principle iff for all BAFs $\mathcal{F} = \langle Ar, att, sup \rangle$, for every extension $E \in \sigma_i^x(\mathcal{F})$, if arguments $a, b \in E$ and a supports b, then $E \in \sigma_i^x(Ar, att, sup \setminus \{(a, b)\})$.*

Principle 4 states that if argument a is not in an extension E and argument b is in this extension E such that a supports b, then E is still an extension after we remove the support from a to b.

Principle 4 (External Support Removal Robustness) *A semantics σ_i^x for BAFs satisfies the external support removal robustness principle iff for all BAFs $\mathcal{F} = \langle Ar, att, sup \rangle$, for every extension $E \in \sigma_i^x(\mathcal{F})$, if argument $a \in Ar \setminus E$ supports argument $b \in E$, then $E \in \sigma_i^x(Ar, att, sup \setminus \{(a, b)\})$.*

Principle 5 and Principle 6 both concern the closure under the support relation. Closure says that if an argument is in an extension, the arguments it supports are also in the extension, while inverse closure says the opposite, i.e. if an argument is in an extension, the arguments supporting it should also be in the extension [8,15,33].

Principle 5 (Closure) *A semantics σ_i^x for BAFs satisfies the closure principle iff for all BAFs $\mathcal{F} = \langle Ar, att, sup \rangle$, for every extension $E \in \sigma_i^x(\mathcal{F})$, if $(a, b) \in sup$ and $a \in E$, then $b \in E$.*

Principle 6 (Inverse Closure) *A semantics σ_i^x for BAFs satisfies the inverse closure principle iff for all BAFs $\mathcal{F} = \langle Ar, att, sup \rangle$, for every extension $E \in \sigma_i^x(\mathcal{F})$, if $(a, b) \in sup$ and $b \in E$, then $a \in E$.*

Principle 7 reflects the idea that if there is no support relation, the extensions under semantics σ_i^x are equivalent to the ones in Dung semantics.

Principle 7 (Extension Equivalence) *A semantics σ_i^x for BAFs satisfies the extension equivalence principle iff for all BAFs $\mathcal{F} = \langle Ar, att, sup \rangle$, $\sigma_i^x(Ar, att, \emptyset) = \sigma_0^x(Ar, att, \emptyset)$.*

Principle 8 and Principle 9 both state the positive effect of supports on the supported arguments. We first present the definition of the status of arguments as introduced by Baroni and Giacomin [3]. Extension-based semantics classifies arguments into three statuses, namely sceptically accepted, credulously accepted, and rejected.

Definition 11. ***(Status of an Argument [3])*** *Let $\mathcal{F} = \langle Ar, att, sup \rangle$ be a BAF. If the set of extensions is empty, all the arguments are declared to be rejected. Otherwise, we say that an argument is: (1) sceptically accepted if it belongs to all extensions; (2) credulously accepted if it is not sceptically accepted and it belongs to at least one extension; (3) rejected if it does not belong to any extension.*

Gargouri et al. [23] write Status$(a, \mathcal{F}) = sk(resp.\ cr, rej)$, and they define the order $\leqslant$ on the set of statuses as expected: $sk > cr > rej$. We denote the set of sceptically accepted (resp. credulously accepted, rejected) arguments of a BAF by $Sk(Ar, att, sup)$ (resp. $Cr(Ar, att, sup)$, $Rej(Ar, att, sup)$. Principle 8 states that adding supports to arguments does not change their status into a lower order. Gargouri et al. [23] call this monotony, but we prefer to use a more specific name (i.e. monotony of status) to make it more precise and avoid ambiguity.

Principle 8 (Monotony of Status) *A semantics σ_i^x for BAFs satisfies the monotony of status principle iff for all BAFs $\mathcal{F} = \langle Ar, att, sup \rangle$, for every extension $E \in \sigma_i^x(\mathcal{F})$, for all $a, b \in Ar$, we have Status $(a, \langle Ar, att, sup \rangle) \leqslant Status(a, \langle Ar, att, sup \cup \{(b, a)\} \rangle)$.*

Principle 9 shows a skeptically accepted argument stays skeptically accepted when supports are added [25].

Principle 9 (Extension Growth) *A semantics σ_i^x for BAFs satisfies the extension growth principle iff for all BAFs $\mathcal{F} = \langle Ar, att, sup \rangle$, for every extension $E \in \sigma_i^x(\mathcal{F})$, it holds that $Sk(Ar, att, sup) \subseteq Sk(Ar, att, sup \cup sup\prime)$.*

Directionality is introduced by Baroni, Giacomin, and Guida [4]. It reflects the idea that we can decompose an argumentation framework into sub-frameworks so that the semantics can be defined locally. For the directionality principle, they first introduce the definition of an unattacked and unsupported set.

Definition 12 (Unattacked and unsupported arguments in BAF). *Given a BAF $\mathcal{F} = \langle Ar, att, sup \rangle$, a set U is unattacked and unsupported if and only if there exists no $a \in Ar \backslash U$ such that a attacks U or a supports U. The set unattacked and unsupported sets in $\mathcal{F}$ is denoted $US(\mathcal{F})$ (U for short).*

Principle 10 (BAF Directionality) *A semantics σ_i^x for BAFs satisfies the BAF directionality principle iff for every BAF $\mathcal{F} = \langle Ar, att, sup \rangle$, for every $U \in US(\mathcal{F})$, it holds that $\sigma_i^x(\mathcal{F}_{\downarrow_U}) = \{E \cap U | E \in \sigma_i^x(\mathcal{F})\}$, where $\mathcal{F}_{\downarrow_U} = (U, att \cap U \times U, sup \cap U \times U)$ is a projection, and $\sigma_i^x(\mathcal{F}_{\downarrow_U})$ are the extensions of the projection.*

Table 1 compares the semantics with respect to the principles. For the defence-based semantics, semantics$_1$ and semantics$_2$ can be classified by the same principles, and they can be distinguished from semantics$_3$ by Principles 3, 7 and 9. Semantics$_4$

and semantics$_5$ are selected from semantics$_0$, they can be distinguished by Principle 3 and Principle 4. However, Table 1 indicates it is not the case that if semantics$_0$ satisfies a principle implies semantics$_4$ and semantics$_5$ also satisfy it, e.g. the results regarding Principle 10. Reduction-based semantics can be distinguished from others by Principles 1, 5, 6 and 8. More precisely, they themselves can be further distinguished by Principle 5 and 6, and surprisingly, only semantics$_7$ satisfies Principle 8. One thing worth noting is that, in the literature, there are two other reductions based on necessary interpretation of support, i.e. one introduces only secondary attacks and the other introduces only extended attacks. Both of them do not satisfy directionality [42]. However, the result in this paper shows when the necessary reduction induces both secondary and extended attacks, semantics$_7$ (except for stable$_7$) satisfy directionality.

Table 1: Comparison of semantics and principles. We refer to the semantics as follows: complete ($\mathbb{C}$), grounded ($\mathbb{G}$), preferred ($\mathbb{P}$) and stable ($\mathbb{S}$). When a principle is never satisfied by a certain reduction for all semantics, we use the $\times$ symbol. P1 refers to Principle 1, and the same holds for the others.

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
σ_0^x	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\times$	$\times$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\mathbb{CGP}$
σ_1^x	$\times$	$\times$	$\times$	$\mathbb{CGPS}$	$\times$	$\times$	$\times$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\mathbb{CGP}$
σ_2^x	$\times$	$\times$	$\times$	$\mathbb{CGPS}$	$\times$	$\times$	$\times$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\mathbb{CGP}$
σ_3^x	$\times$	$\times$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\times$	$\times$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\times$	$\mathbb{CGP}$
σ_4^x	$\times$	$\mathbb{CGPS}$	$\times$	$\mathbb{CGPS}$	$\times$	$\times$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\times$
σ_5^x	$\times$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\times$	$\times$	$\times$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\times$
σ_6^x	$\mathbb{CGPS}$	$\times$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\times$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\times$	$\times$
σ_7^x	$\mathbb{CGPS}$	$\times$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\times$	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\times$	$\times$	$\mathbb{CGP}$

4 Related Work

The notion of support has drawn the attention of many scholars in argumentation theory, including the role of support in argumentation, whether attack and support should be treated as equals, the link between the abstract approaches and ASPIC+, and also higher-order abstract bipolar argumentation frameworks [36,37,24,11]. We now review and comment on the three approaches to define semantics studied in this paper. For the defence-based approach, we adapted the core notions in Dung's theory. There are other variants of semantics that adapt these notions, such as weak defence for weak admissibility semantics [7,20], but it is not related to the notion of support. For selection-based approach, semantics$_4$ and semantics$_5$ select extensions based on the number of internal (or external) supports received respectively. Such an approach has already been used in some previous work, and most of them are based on preference [2,25] or weight of arguments and relations [19,26]. More recently, Gargouri et al. proposed an approach to select the best extensions to BAFs by comparing the number of received supports with scores for each extension [23]. The reduction-based approach allows a BAF to be transformed into an argumentation graph that has been already discussed in the literature [16,30,36,11]. There is a striking similarity at the abstract level between support in bipolar argumentation and preference-based argumentation, as both can be seen as reductions, as well as both can be used to select extensions [25]. For other approaches to bipolar argumentation semantics, Cayrol et al. proposed some properties of gradual semantics for bipolar argumentation [12], after which Evripidou and Toni provided a concrete definition of gradual semantics for bipolar argumentation [22] and introduced the quantitative argumentation debate (QuAD) framework [6]. Concerning aggregating bipolar argumentation frameworks, Chen considered how to cope with different opinions on support relations and analyse which properties can be preserved by desirable aggregation rules during aggregation of support relations [17]. Lauren et al. also considered aggregating bipolar assumption-based argumentation frameworks under the assumption that agents propose the same set of arguments, different sets of attacks and different interpretations of supporting arguments [28].

Baroni and Giacomin are the first to adopt a principle-based approach for classifying argumentation semantics [3], which was followed by other papers axiomatising abstract argumentation [40], preference-based argumentation [25] and agent argumentation [41]. There are papers that propose principles for bipolar ranking-based/gradual semantics [1], and their generalisations [5]. However, there is a lack of such work for extension-based semantics. Cayrol et al. compared bipolar argumentation semantics, they discussed the semantics based on deductive and necessary interpretations, and provided a few properties, e.g. closure, coherence and safe [16]. Inspired by this work, Yu and van der Torre analysed reduction-based semantics with more properties [42], however, they have only considered reduction-based semantics, without comparing them with others.

5 Summary and Future Work

In this paper, we gave an axiomatic analysis of bipolar argumentation semantics. We considered three approaches, namely defence-based, selection-based, and reduction-based approaches. In total, we introduced seven different types of semantics and studied them together with Dung semantics, which is the baseline and does not take into account supports. Semantics$_{1-3}$ are defence-based, i.e. they are defined by generalising the new notions of defence. Such an approach allows us to treat attack and support at the same level. Semantics$_4$ and semantics$_5$ are not only based on admissibility, but also borrow the idea from another field, i.e. social voting, to use the number of supports as a way of voting or selecting to derive extensions. Semantics$_6$ and semantics$_7$ are based on the notions of necessary and deductive support respectively. We evaluated those semantics against the set of ten principles. The results are shown in Table 1. Given the diversity of interpretations of support, such axiomatic analysis can provide us an overview and systematic assessment of different approaches. It can help us to choose a semantics for a given task or a particular application in function of the desirable properties. One can look at the table and see if there exists a semantics that satisfies the given desiderata.

An interesting question for future work is how to relate semantics defined by various approaches, e.g. can we define a new defence with attacks and supports indicating the deductive, necessary or evidential interpretation of support? We have semantics$_2$ stating that only a supported argument can defend others, which also reflects the idea of evidential support [30,34]. In this paper, we use dynamic properties, e.g. the robustness of semantics when adding and removing support. This could be further developed by analyzing labelling-based semantics of bipolar argumentation. The distinction between arguments labelled out and undecided makes the principles more precise. We also consider that the approaches to the dynamics of argumentation can be used as a source for principles [9,25]. Another possible direction is to study the relation between the principles, for example, to verify whether one principle implies another one, or if there is a set of principles such that no semantics satisfies all of them. Lastly, in the same spirit of this paper, another future work is the principle-based analysis of higher-order bipolar argumentation frameworks [27].

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